Patterns
Session 2

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Overview of Patterns	Overview of Patterns	52			
Exploring Various Types of Patterns	I Need A Necktie, Please!	54	1.21, 2.25, 3.24, 4.21	I Need a Necktie, Please!	Pattern blocks, adding machine tape strips, paper pattern blocks
	Up, Up and Away!	57	1.21, 2.25, 3.24, 4.21	Up, Up, and Away!	Linking cubes, graph paper, adding machine strip tape
	How High Are My Castle Walls?	59	1.21, 2.25, 3.24, 4.21	How High Are My Castle Walls?	Wooden pattern blocks, adding machine tape, paper pattern blocks
	Exactly How Many Doors Are We Talking About?	61	3.24, 4..21, 5.20	Exactly How Many Doors Are We Talking About?	Wooden pattern blocks
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Patterns in Nature	The Jeweled Snake	70	K.18, 1.21, 2.25, 3.24, 4.21	The Story of The Jeweled Snake	Pattern blocks, overhead pattern blocks
	Fibonacci Numbers	74	K.18, 1.21, 2.25, 3.24, 4.21, 5.20	Patterns and Relationships	Examples or pictures of Fibonacci Numbers, tree branches with leaves, pinecones, pineapple, sunflower or daisy

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Patterns on the Hundreds Board	Grid Pictures	77	1.21, 2.25, 3.24, 4.21	Picture This, Grid Pictures	Linking cubes or small colored squares
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	Patterning on the 100 Chart	85	1.21, 2.25, 3.24, 4.21, 5.20	Patterning on the 100 Chart	Linking cubes, recording sheet
	Odd and Even	90	2.5		Color tiles
	Line Up	94	4.21	Line Up! Activity Sheet	Meter sticks, yarn, and clothes pins
	Arrow Math	96	1.21, 2.25, 3.24, 4.21, 5.20	Arrow Math	
Patterns and Literature	The King's Commissioner	99	3.24, 4.21, 5.20		*The King's Commissioners*, 100s boards, centimeter cubes, crayons or markers
	The King's Chessboard	101	3.24, 4.21, 5.20		*The King's Chessboard*, 100s boards, grains of rice

Activity: Overview of Patterns

The Virginia Standards of Learning state that students will be able to create, recognize, describe, analyze and extend a variety of patterns. This instructional module focuses on the mathematics of specific types of patterns and presents staff development activities to engage teachers in further developing their knowledge of patterns that can inform their instructional practices. Problem-solving activities are intended to engage participants in learning through the use of manipulatives, through the discovery and development of strategies, through the use of thought-provoking questions, and through activities where patterning can be found in real-world situations. Attention should be given to strategies for identifying and generalizing patterns. Patterns should be represented and modeled in a variety of ways including numerical, geometric, graphic, and algebraic.

Since the physical world is full of patterns, students enjoy finding them and understanding them. Children often have questions about patterns such as those found in nature: “Is there a pattern to the number of petals on flowers?”, “Why do all snowflakes have six sides?”, ” Why do all pineapples have the same number of clockwise and counterclockwise spirals?” As teachers, it is important to help students to learn about patterns through observation, experimentation, and discovery and help them to apply this understanding to learn mathematics. Opportunities to understand the various types of patterns and experience the wonder of patterns should be an objective of the elementary mathematics curriculum.

Repeating Patterns

The simplest types of patterns are **repeating patterns**. The patterns can be oral, such as the refrain in “Old MacDonald’s Farm” -- “e-i-e-i-o”; or physical with clapping and snapping patterns; or combinations of both with songs like the “hokey pokey”. In each case students need to identify the basic unit of the pattern and repeat it. Opportunities to create, recognize, describe and extend repeating patterns are essential to the primary school experience.

Sample Repeating Patterns (i.e., repeating the basic unit)

1. ABABABAB
2. ABCABC
3. AABBAABBAABB
4. AABAAB
5. AABCAABC
6. ABACABAC

Growing Patterns

Students find that **growing patterns** are more difficult to understand than repeating patterns, as they must not only determine what comes next, but they must also begin the process of generalization. Students need experiences with growing patterns both in arithmetic and geometric formats.

Sample Growing Patterns (i.e., where one variable changes in the basic unit)

1. ABAABAAABAAAAB
2. ABABBABBBABBBB
3. ABCAABCAAABC

Arithmetic Patterns

In **arithmetic patterns**, also called arithmetic sequences, students must determine the difference, called the "common difference", between each succeeding number in order to determine what is added to each previous number to obtain the next number.

Sample Arithmetic Patterns
6, 9, 12, 15, 18, ...
5, 7, 9, 11, 13, ...
16, 12, 8, 4, 0, -4, -8 ...

A generalization for these **arithmetic patterns** could be expressed as follows:

□, □ + Δ, (□ + Δ) +Δ, (□ + Δ + Δ) +Δ, (□ + Δ + Δ + Δ) +Δ, (□ + Δ + Δ + Δ + Δ) +Δ, …

Geometric Patterns

In **geometric patterns**, students must determine what each number is multiplied by to obtain the next number in the geometric sequence. This multiplier is called the "common ratio".

Sample Geometric Patterns
2, 4, 8, 16, 32, ...
1, 5, 25, 125, 625, ...
80, 20, 5, 1.25, ...

A generalization for these **geometric patterns** could be expressed as follows:

□, □* Δ, (□ * Δ) * Δ, (□ * Δ * Δ) * Δ, (□ * Δ * Δ * Δ) * Δ, (□ * Δ * Δ * Δ * Δ) *Δ, …

Activity: I Need A Necktie, Please!

Format: Small Group

Objective: Participants will experience rhythmic patterns, describe those same patterns using wooden pattern blocks, and finally copy the patterns onto paper as they create a necktie for Father Giraffe.

Related SOL: 1.21, 2.25, 3.24, 4.21

Materials: Pattern blocks, adding machine tape strips, paper pattern blocks, I Need A Necktie, Please! Activity Sheet

Time Required: 20 minutes

Directions:

1. Engage participants in the rhythmic pattern clap, clap, pat your lap, clap, clap, pat your lap, and so on.
2. Ask participants to create a pattern with their pattern blocks that is the same as the clap, clap, pat pattern such as red, red, blue, red, red, blue and so on.
3. Repeat this activity several times with different rhythmic patterns such as snap, clap, snap, clap until all participants have created a pattern with their pattern blocks.
4. Distribute adding machine tape strips and explain to participants that they are to copy their pattern onto the strips with paper pattern blocks because they are going to make Father Giraffe a special necktie with a pattern design.
5. Remind participants that they are to repeat their pattern over and over until they have filled Father's necktie from top to bottom.
6. Instruct participants to paste their neckties on Father Giraffe.
7. When participants have completed their neckties, explain that they are now going to display their ties for all to see.
8. Discuss the different patterns and the numerous ways to sort and organize the patterns on the neckties such as ABB, AABB, AB, or even ABCD.

Patterns

Repeating Patterns (Repeating the basic unit)

1. ABABABAB
2. ABCABC
3. AABBAABBAABB
4. AABAAB
5. AABCAABC
6. ABACABAC

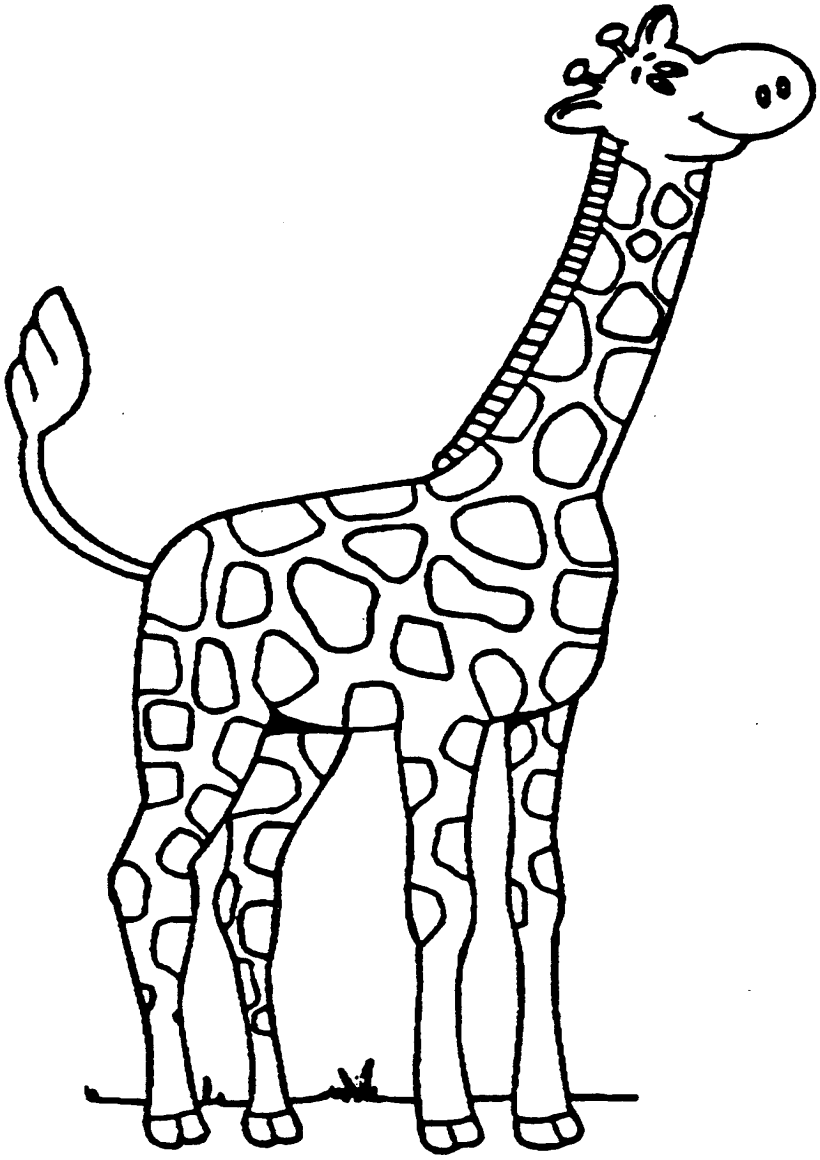
Growing Patterns

1. ABAABAAABAAAAB
2. ABABBABBBABBBB
3. ABCAABCAAABC

Other Patterns - Unexpected, Unusual

1. Seasons
2. Nature
3. Honeycombs
4. Stop Light - G,Y,R, G,Y,R
5. Class Schedules
6. Music
7. Rhythm -- clapping / soft-loud

I Need A Necktie, Please! **Name:** _____



Activity: Up, Up and Away!

Format: Small Group

Objective: Participants will create a repeating pattern with linking cubes and interpret that same pattern with rhythmic gestures. In turn, those patterns will be identified according to a letter value such as ABAB or ABBA. Participants will then be asked to transcribe that pattern onto adding machine tape with paper pattern blocks to create the tail on a kite.

Related SOL: 1.21, 2.25, 3.24, 4.21

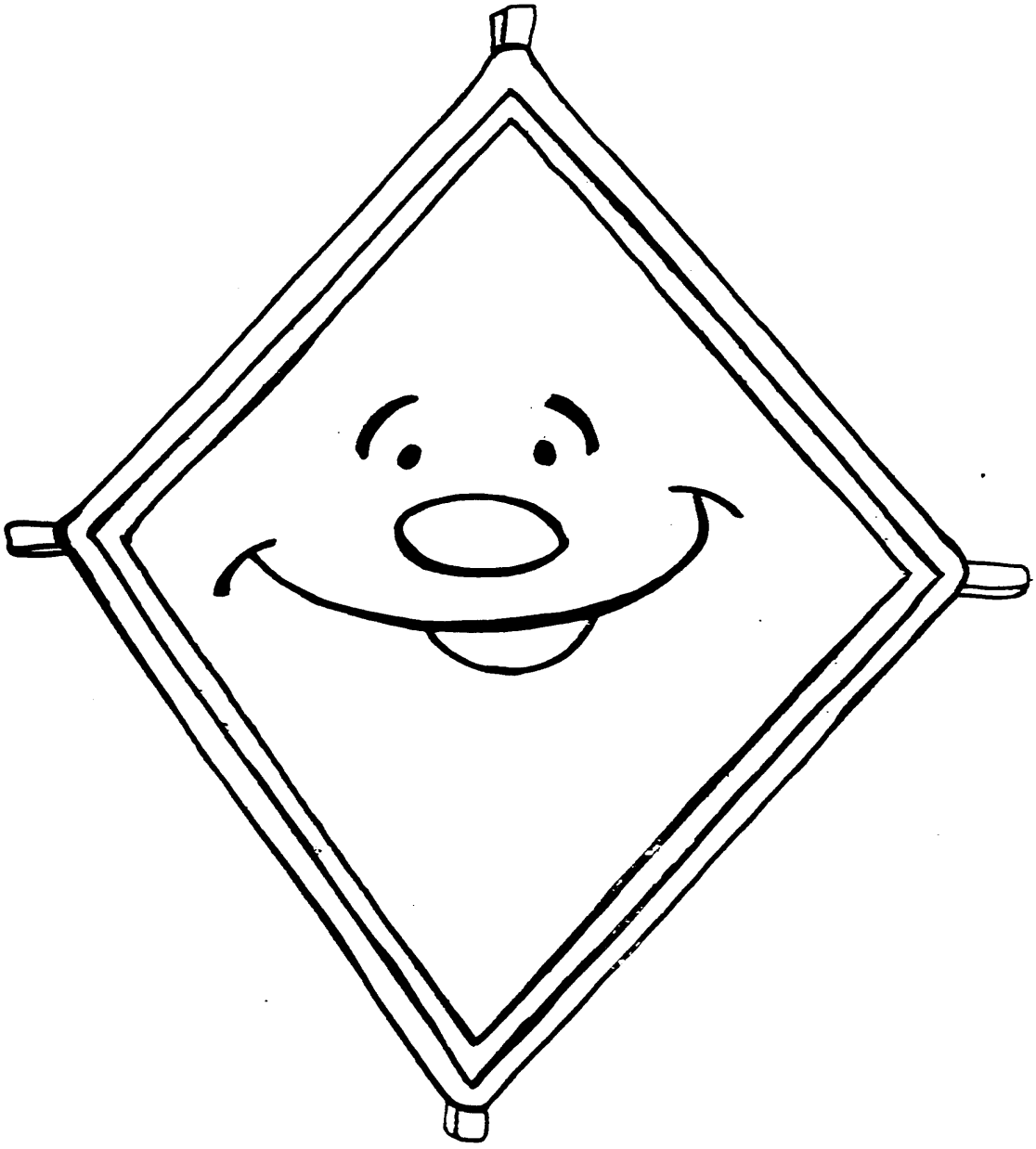
Materials: Linking cubes, graph paper, adding machine strip tape, Up, Up, and Away! Activity Sheet

Time Required: 20 minutes

Directions:

1. Ask participants to make a repeating pattern with two colors of linking cubes.
2. Explain to participants that they should make two or three repeats of the pattern with the cubes.
3. Discuss the different patterns participants have made with their linking cubes. Interpret each participant's pattern by snapping or clapping, by saying the colors aloud, or by playing touching games. The pattern ABB can be interpreted snap, clap, clap, snap, clap, clap and so on. The pattern ABAB could be interpreted red, blue, red, blue, red, blue, red, blue, and so on. An AABB pattern could be interpreted touching nose, nose, shoulders, shoulders and so on, and an ABAB could be interpreted touching nose, shoulders, nose, shoulders, nose, shoulders, and so on.
4. As each participant's pattern is interpreted, be sure to emphasize that the pattern continues on beyond the number of linking cubes.
5. Have the participants record their patterns by coloring the pattern they have created on graph paper. Participants should color two to three feet of their pattern on the graph paper and then paste their completed pattern on adding machine tape in order to create a kite tail.
6. Instruct the participants to cut out the kite and then paste on the completed patterned tail.

Up, Up and Away! **Name:** _____



Activity: How High Are my Castle Walls?

Format: Small Group

Objective: Participants will investigate linear patterns using concrete objects as they construct castle walls to encompass a ready-made castle.

Related SOL: 1.21, 2.25, 3.24, 4.21

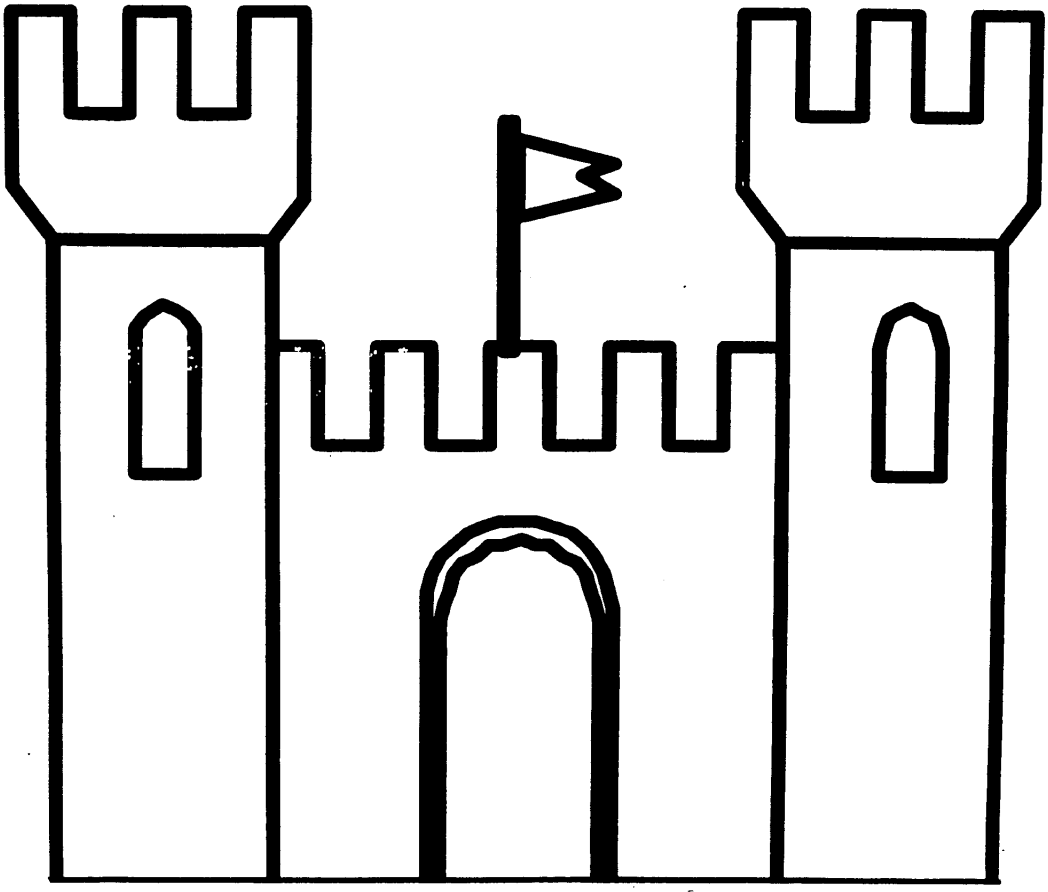
Materials: Wooden pattern blocks, adding machine tape, paper pattern blocks, How High Are My Castle Walls? Activity Sheet

Time Required: 20 minutes

Directions:

1. Have participants choose pattern blocks and create a wall with a repeating pattern around the castle.
2. Encourage participants to discuss the different pattern walls that they create and the names of the different colors or shapes that they use to create their castle wall such as square, triangle, triangle, square, triangle, triangle and so on.
3. Instruct participants to copy their pattern walls onto adding machine tape with paper pattern blocks. Remind them to repeat their pattern over and over.
4. Ask participants to tape the ends of their pattern walls together to form a circle around the castle.
5. Participants should color and cut out the castle, roll it into a cylinder and secure it in a standing position. Conclude the activity by placing participants' paper pattern block wall around their castles.

How High Are my Castle Walls? **Name:** _____



Activity: Exactly How Many Doors Are We Talking About?

Format: Small group

Objective: Participants will use concrete objects to construct "walls" and then determine how many "doors" are located along the wall. Participants will be able to develop a number pattern and use that pattern to look for a rule to predict the number of blocks.

Related SOL: 3.24, 4.21, 5.20

Materials: Pattern blocks, Exactly How Many Doors Are We Talking About? Activity Sheet

Time Required: 20 minutes

Directions:

1. Instruct the participants to create a wall of pattern blocks. Each section of the wall will contain a sequence of one red trapezoid, one green triangle, and one red trapezoid. A wall with one green door will take three blocks to build.
2. Ask the participants to add more blocks to increase the size of this wall. The expanded wall with two green doors will take five blocks to build.
3. Explain that the wall will continue with the same pattern. Challenge the participants to compute the total number of pattern blocks in the wall when there are fifty green doors. {(50 x 2) +1 = 101}
4. Participants will not have enough blocks to actually build the entire wall so encourage them to fill out the chart in order to generalize the number pattern and the rule the wall represents. {(Doors x 2) + 1 = total blocks}
5. Have participants write their answers on the recording sheet and write about their findings.

Name: _____

Exactly How Many Doors Are We Talking About?

Number of Doors	Total Number of Blocks
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
|	
|	
|	
50	

Activity: Building Staircases

Format: Small Group

Objective: Participants will determine how many centimeter cubes will be needed to build a staircase with one hundred steps. Participants will be able to extend a given pattern using concrete manipulatives to create a staircase with one hundred steps.

Related SOL: 4.21, 5.20

Materials: Centimeter cubes, Building Staircases Activity Sheet

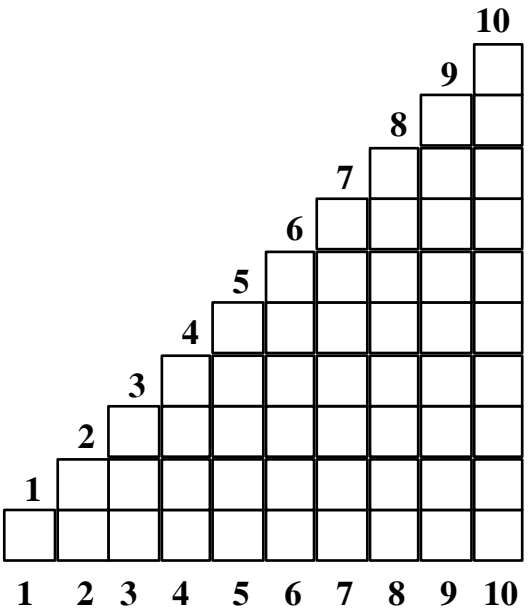
Time Required: 20 minutes

Directions:

1. On the overhead projector, the teacher should build a staircase, beginning with one step and continuing until four steps have been created.
2. As you continue to increase the staircase pattern, ask participants, "In this staircase pattern, which step is this? How many centimeter cubes does the pattern have so far? How many centimeter cubes will we need to add to make the next step?"
3. Instruct the participants to work with their partner using centimeter cubes to build staircase models.
4. Encourage participants to record the patterns they begin to see. Ask the participants: "How many centimeter cubes will you need to build a staircase with one-hundred steps?" Help participants explore different possibilities for finding the correct answer.

Building Staircases

Name: _____



Step	Total Number of Cubes
1	
2	
3	
4	
5	
.	
.	
100	
n	

Activity: Tons of Tunnels

Format: Small Group

Objective: The participant, using concrete materials, will be able to identify the missing pattern pieces that are hidden from view.

Related SOL: K.18, 1.21, 2.25, 3.24

Materials: Linking cubes, top half of cardboard tubes from paper towels, Tons of Tunnels Activity Sheet

Time Required: 10 minutes

Directions:

1. Direct participants to make a ABA, AABB, ABAC, ABC or even a ABCA pattern using linking cubes. Encourage the participants to repeat their patterns at least three times until they have used 12 linking cubes.
2. Ask participants to call this strip of repeating pattern with the linking cubes a pattern strip.
3. Model for the class how parts of the pattern can be hidden by placing a cardboard tube over a section of the pattern strip.
4. Ask participants to work with a partner and hide a section of their pattern by covering it with the cardboard tube they are provided.
5. Instruct the participants to work with their partner and guess what is hidden under the tunnel.
6. Allow participants to check their predictions by removing the cardboard tube.
7. Reverse the process and allow the partner to build the pattern strip and hide the pattern with the cardboard tube.

Tunnels Name: _____

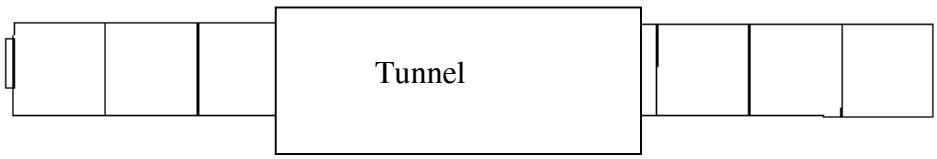
Materials Needed: Linking cubes and top half of cardboard tubes

1. Make an ABA, AAB, AABB, AABC, ABAC, ABC or an ABCA pattern using linking cubes. After you select your pattern unit, repeat it at least 3 times until you use 12 linking cubes. This is called your pattern strip.

2. Place the cardboard tube over part of your pattern strip.

3. Find a partner and ask him/her to guess what is under the tunnel. Let them check the prediction by lifting the tunnel.

4. Repeat steps 1-3 letting your partner make and hide the pattern strip.



Activity: How Many Beads are Hidden Under that Cloud?

Format: Small Group

Objective: The participant will be able to determine how many beads are hidden under the cloud.

Related SOL: 4.21. 5.20

Materials: Colored beads, string or yarn, needles, How Many Beads Are Hidden Under The Cloud? Activity Sheet

Time Required: 10 minutes

Directions:

1. Invite participants to create a growing pattern by stringing two colors of beads.
2. Encourage participants to repeat their growing patterns at least six times.
3. Such a pattern could then be identified as an ABBAABBBBAAABBBBBBBB pattern.
4. Ask for volunteers to explain what the A pattern and the B pattern are (doubling).
5. Help participants to see that the A pattern and the B pattern are growing at different rates.
6. After the participants are comfortable with the doubling effect of the A and B patterns they have created with their colored beads, direct that they work with a partner and cover a section of their beaded patterns from view.
7. Allow time for each team to reverse roles and determine the number of missing pattern pieces.
8. Invite whole group discussion and inquiry in order to determine if team understanding is adequate.
9. Display the “How Many Beads are Hidden under the Cloud?” Activity Sheet on the overhead projector.
10. Ask participants to again work with a partner to determine the number of missing beads.

ANSWER:

Using the symbols: Black Bead = B White Bead = W

The sequence would proceed as follows:
1B, 2W, 2B, 4W, 3B, 8W, 4B, 16W, 5B, 32W, 6B, 64W, 7B. ...

What's Missing?

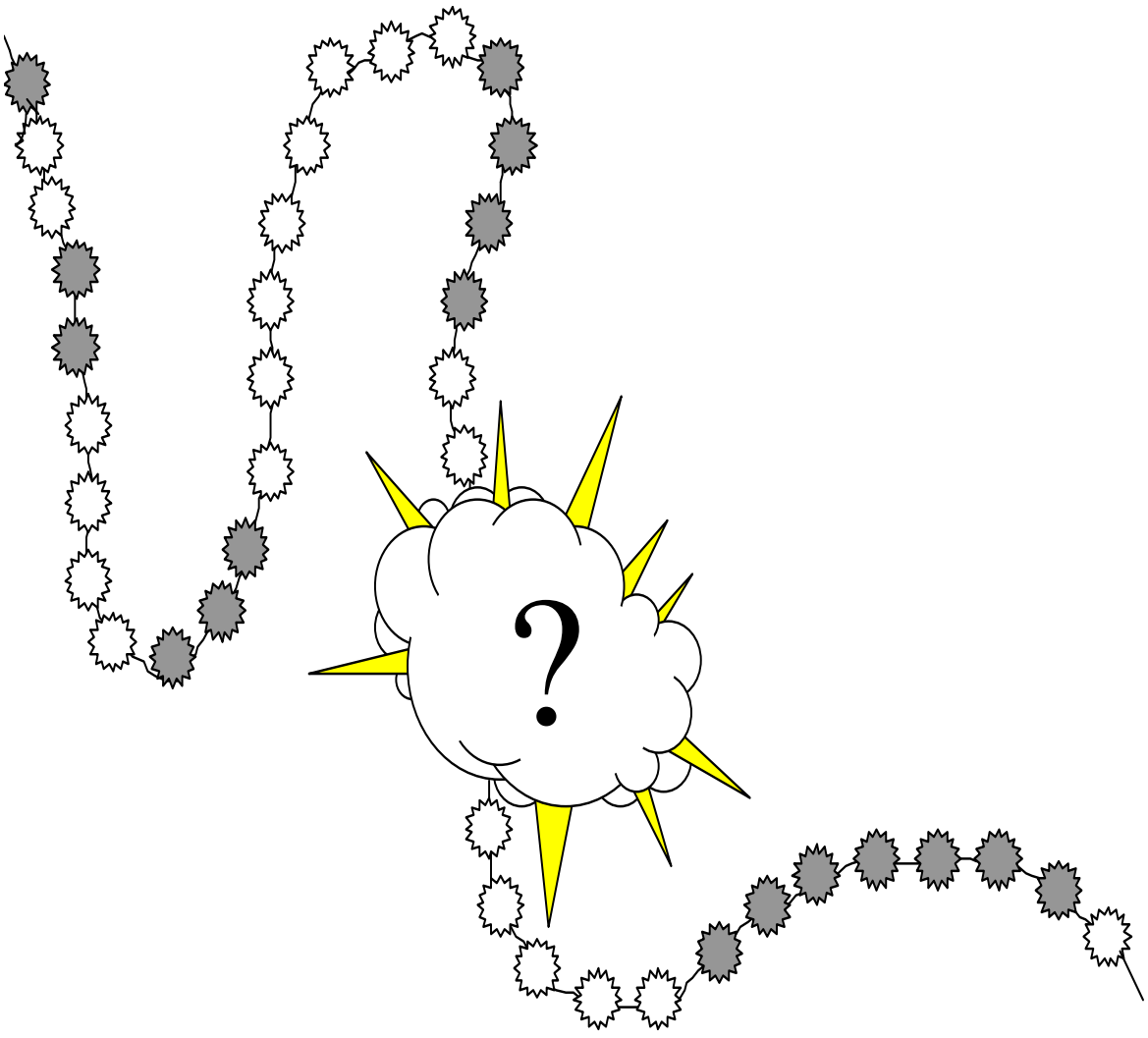
Of the 16 White Beads - 2 are showing and 14 are under the cloud
Then 5 Black Beads are under the cloud
Then 32 White Beads are under the cloud
Then 6 Black Beads are under the cloud

Of the next 64 White Beads, 5 are showing, so 59 are under the cloud

Under the Cloud:
14 + 5 + 32 + 6 + 59 = 116 are hidden under the cloud

Clouds **Name:** _____

How Many Beads are Hidden Under the Cloud?



Activity: The Jeweled Snake

Format: Whole Group

Objective: Participants will experience an integration of literature and mathematics when they are asked to create twelve linear and geometric patterns with paper pattern blocks and actual pattern blocks after listening to the selection, "The Jeweled Snake". Participants will be able to recognize, describe, and extend a repeating pattern using pattern blocks as the manipulative.

Related SOL: K.18, 1.21, 2.25, 3.24, 4.21

Materials: Story and picture of the "Jeweled Snakes" Activity Sheet, wooden pattern blocks for participants, overhead plastic pattern blocks for instructor

Time Required: 20 minutes

Directions:

1. Identify for participants the various pattern block shapes: yellow hexagons, blue rhombuses, red trapezoids, green triangles, orange squares, and tan parallelograms.
2. Read the selection "Jeweled Snakes" to the participants and give them the handout of the story.
3. Model on the overhead projector with overhead pattern blocks the Father Snake's pattern, "Yellow hexagon and blue rhombus shapes that fit together".
4. Model on the overhead projector with overhead pattern blocks Mother Snake's pattern, "Yellow hexagon shapes with red trapezoids in between".
5. Distribute the picture of the Jeweled Snake and ask the participants to use pattern blocks to make the repeating patterns of the ten remaining snakes.
6. Have various sets of partners model for the class the different patterns they have created for each member of the snake family.

Story of the Jeweled Snakes

At the back of Farmer Max's garden was a deep, dark hole where a family of snakes lived. As snake families go, it was a medium-sized family. In all, there were 12 snakes: a father, a mother, their 6 children, a grandmother and a grandfather, and an aunt and an uncle. They all lived together in their little snake home.

There was something very special about these snakes. Their skin was covered with colorful shapes that looked like bright colored jewels running along their backs in beautiful repeating patterns. One shape that all of the snakes had in common was a hexagon.

The father snake was just about a foot long. He was covered with yellow hexagon and blue rhombus shapes that fit together in a long row. The shapes ran all the way down his back, one colorful shape after another. Now, the mother snake was covered with yellow hexagons, too. But instead of blue shapes, there were red trapezoids in between. Both snakes were beautiful.

In fact, each snake looked different:

* The grandfather snake was covered with yellow, red, and green shapes.
* The grandmother snake was covered with yellow, blue, and green shapes.
* The uncle snake was covered with yellow, orange, and tan shapes.
* The aunt snake was covered with yellow, blue, and tan shapes.
* Each of the children snakes was covered with yellow, blue, and red shapes. But they each looked different.

One day the snakes were going out looking for food, just as the father snake slithered from the hole, Max caught sight of him. "What a lovely snake," Max thought. "The pattern looks like jewels." Max decided the pattern would be perfect to put on the wood frame he was making for a picture. So he carefully studied the way that the blue shapes and yellow hexagons fit together, and he made a copy with some pattern blocks. Then he went to get his wife, Cleo.

By that time, the father snake was gone and the mother snake was just inching out of the hole. But Max didn't notice that the snake he was looking at now was a different snake. His wife, Cleo, said, "That snake is beautiful, but it doesn't look like the pattern you made." "I don't understand it," said Max. "I thought I was being so careful when I copied the pattern."

"Oh, well," said Cleo. "Let's copy the pattern and make another frame." So they did. Then they went to get their daughter, Josephine.

By that time, the mother snake was gone and the oldest snake child was just coming out of the hole. But Max, Cleo, and Josephine didn't know that. Josephine said, "That's a beautiful snake, but it doesn't look like either of the patterns you made."

"You're right, daughter!" replied Max. "Well, let's copy this one, too."

Max, Cleo, and Josephine copied the patterns of the 12 snakes that came out of the hole, one by one. Each time they saw a different snake. So each time the pattern was different. They never did figure it out. But you should see all the beautiful frames they made.

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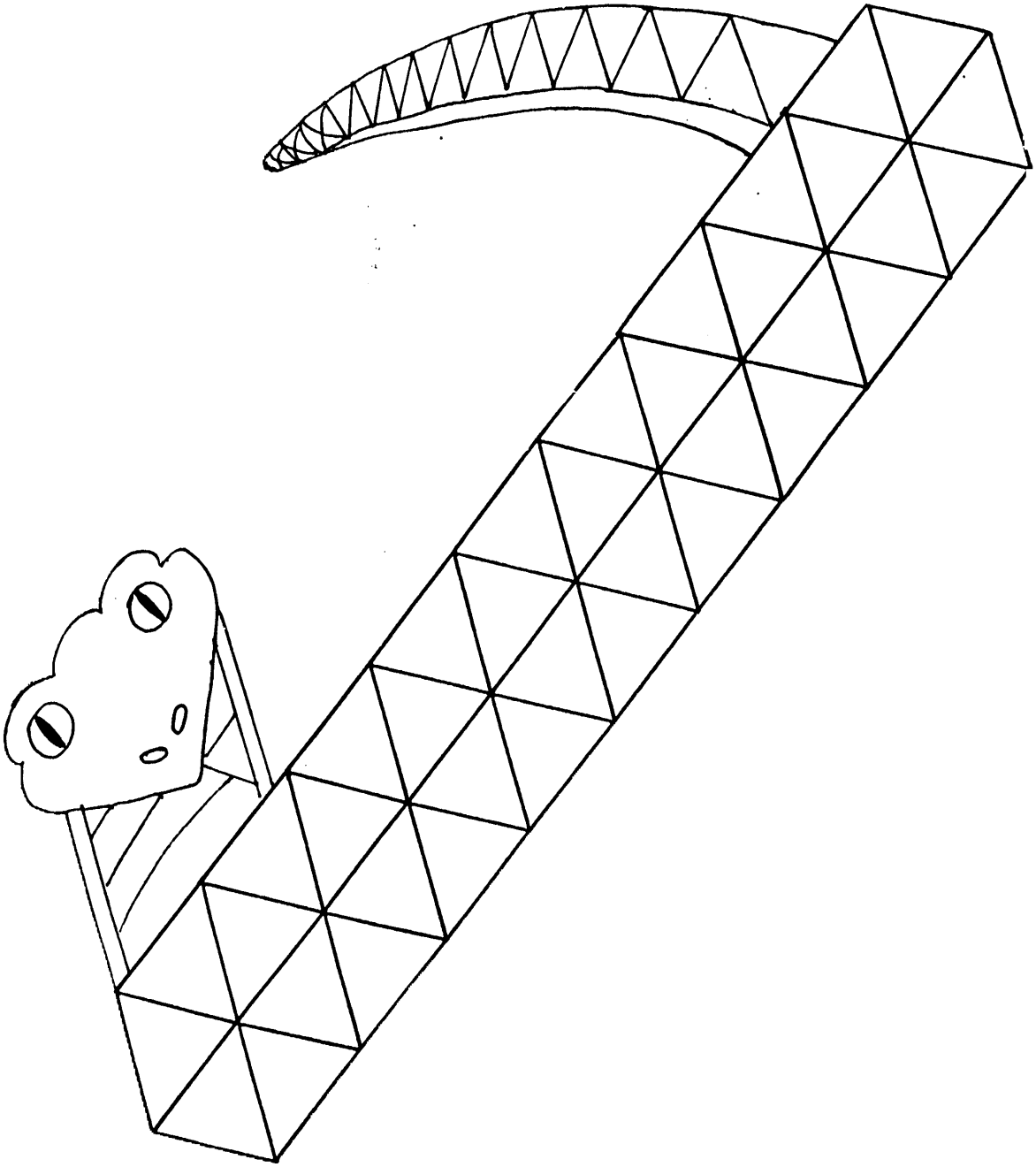
The Jeweled Snake

Name: _____



Activity: Fibonacci Numbers

Format: Whole Group

Objective: Participants will be able to create, recognize, describe, analyze, and extend a variety of patterns.

Related SOL: K.18, 1.21, 2.25, 3.24, 4.21, 5.20

Materials: Examples or pictures of Fibonacci Numbers, tree branches with leaves, pinecones, pineapple, sunflower or daisy, Patterns and Relationships Activity Sheet

Time Required: 30 minutes

Directions:

1. Say: *Research and our experiences as teachers continue to tell us that children who comprehend patterns in the world around them acquire and retain information much more quickly than those children who do not see or understand patterns. Those children who never seem to comprehend patterns must reinvent the wheel each and every time. For example, the child who never recognizes that a word ending in the letter "e" usually has a long vowel sound must begin the task of determining long or short vowels anew each time they are questioned because no connections have been made. Today's question is exactly how to help all children recognize and use patterns.*

   Teachers must not start from ground zero because each and every child comes to school with many years of experience in patterning. Each time the sun comes up and each time the sun sets, each time the child's father smiles when happy and frowns when upset, each time a light comes on and each time that light is turned off, and each time the child drops his spoon and it falls to the ground that child experiences patterning. Just learning to talk requires a tremendous grasp of patterning.

   Our job as teachers is to help put children's understandings into words. Our job as classroom teachers is to help students to develop their understated knowledge and name what they already know.

   Some patterns in nature have been identified as Fibonacci Numbers, named after an Italian mathematician named Leonardo de Pisa and nicknamed Fibonacci. Today Fibonacci is best remembered for a sequence of numbers that bear his name. This sequence of numbers always begins with 1,1. Each new number is then found by adding the two preceding numbers.

2. Show participants the first six Fibonacci Numbers.

   1, 1, 2, 3, 5, 8, ...

3. Invite participants to work with a partner to determine the next six numbers in the sequence of Fibonacci Numbers.

   1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...

4. Say: *The Fibonacci Numbers describe a wide variety of phenomena in music, art, and nature. The numbers of leaves on many plants are Fibonacci Numbers as well as many branches on certain trees.*

   The numbers of spirals on pineapples or pinecones are Fibonacci Numbers. The center of a daisy and a sunflower has clockwise and counterclockwise spirals that are consecutive Fibonacci Numbers.

5. Encourage participants to investigate other examples of Fibonacci Numbers found in nature.
6. Conclude the concrete investigations by moving the participants to the abstract.
7. Display the Patterns and Relationships Activity Sheet on the overhead projector.
8. Assign partners the task of determining pattern commonalities and pattern differences.

Patterns and Relationships

For each pair, give at least one characteristic that the patterns have in common and at least one way that the patterns are different.

Example: 2, 4, 6, 8, 10 ..... versus 3, 5, 7, 9, 11 ....

Same: Difference of 2 between terms / Increasing

Different: One is even numbers and the other is odd

1. 2, 4, 6, 8, 10 ..... versus 2, 4, 8, 16, 32 .....
   Same:
   Different:

2. 3, 4, 3, 4, 3, 4 .... versus a, b, a, b, a, b .....
   Same:
   Different:

3. 5, 6, 7, 8, 9 ..... versus 9, 8, 7, 6, 5 .....
   Same:
   Different:

4. 5, 10, 15, 20, 25 .... versus 3, 8, 13, 18, 23 .....
   Same:
   Different:

5. 10, 20, 30, 40 ..... versus 100, 200, 300, 400 ....
   Same:
   Different:

Activity: Grid Pictures

Format: Small group

Objective: The participant will become familiar with the 0-99 chart and will be able to identify number patterns and relationships on the chart.

Related SOL: 1.21, 2.25, 3.24, 4.21

Materials: 0 — 99 chart for each participant, linking cubes or small colored squares of paper that will cover a number on the chart

Time Required: 10 minutes

Directions:

1. Distribute Picture This! Activity Sheet and linking cubes to participants.
2. Participants will cover the appropriate number with the indicated color linking cube as the teacher reads each clue.
3. Use the following example or make up other clues. After participants are familiar with this activity, let them create their own picture and a corresponding set of clues. Participants may exchange clue lists to try new problems.

Example 1: FLOWER

Yellow: 11,12,21,22,16,17,26,27, 51,52,61,62,56,57,66,67

Brown: 33,34,35,43,44,45

Green: 54,64,74,84,94,85,76,93,82

Example 2: UMBRELLA

Red: 4,13,15,22,23,24,25,26, 31,32,33,34,35,36,37, 40,41,42,43,44,45,46,47, 48,50,52,54,56,58

Black: 64,74,84,94,82,92,93

Grid Pictures

Example 1: FLAG

Yellow: 0

Brown: 10, 20, 30, 40, 50, 60, 70, 80, 90

Blue: 11, 12, 21, 22

Red: 13, 4, 15, 16, 17, 18, 19, 31, 32, 33, 34, 35, 36, 37, 38, 39, 51, 52, 53,54, 55, 56, 57, 58, 59

White: 23, 24, 25, 26, 27,2 8, 29, 41, 42, 43, 44, 45, 46, 47, 48, 49

Example 2: TREE

Green: 4, 5, 13, 14, 15, 16, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 47, 51, 52, 53, 54, 55, 56, 57, 58, 61, 62, 63, 64, 65, 66, 67, 68

Brown: 74, 5, 84, 85,94, 95

Example 3: HEART (all red)

a. one more than 53
b one less than 8
c. one more than 17
d. one less than 33
e. one more than 13
f one less than 13
g. one less than 48
h. one more than 37
i one more than 20
j. one more than 24
k one less than 4
l. one more than 55
m. one less than 30
n. one less than 44
o. one more than 15
p. one less than 66

Example 4: TURTLE (all green)

a. 5 x 7	b. 4 x 9	c. 5 x 13
d. 3 x 15	e. 2 x 23	f. 1 x 47
g. 53 x 1	h. 4 x 16	i. 11 x 5
j. 8 x 9	k. 3 x 23	l. 4 x17
m. 8 x 7	n. 1 x 57	o. 1x 79
p. 1 x 67	q. 3 x 22	r. 4 x 11
s. 2 x 29	t. 8 x 5	u. 2 x 27
v. 1 x 41	w. 2 x 25	x. 1 x 51
y. 2 x 31	z. 3 x 21	

Example 5: KITE

Red

a. 1 less than 2 dozen
B 3 x 11
c. 3 tens and 4 ones
d. all numbers between 30 and 60 with a 2 in the ones place
e. 2 less than 55
f. 1 greater than 40
g. all numbers between 42 - 46
h. 5 tens and 4 ones
i. 70-7

Yellow

j. 1 less than 75
k. 15 less than 100
l. 100 — 4

Example 6: FLOWER

Yellow

1 ten and 1 one	one more than 50	one less than 68	9 x 7
one more than 51	4 x 3	7 x 3	one less than 62
9 x 3	one more than 56	8 x 2	8 x 7
one more than 21	6 tens and 2 ones	one less than 18	one more than 65

Brown:

3 tens and 3 ones	one more than 33	9 x 5	5 x 7
4 x 11		1 x 43	

Green

9 x 6	1 more than 63	1 less than 75	1 less than 85
9 tens and 4 ones	1 more than 84	7 tens and 6 ones	9 tens and 3 ones
2 more than 80			

PICTURE THIS

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Activity: What Comes Next on the "Picture This" Chart

Format: Small groups and pairs

Objective: Participants will explore patterns on the chart. Putting together a hundred chart puzzle requires participants to consider the place value patterns of one more, one less, and ten more and ten less. Participants will be able to identify the vertical and horizontal relationships on the 0-99 chart.

Related SOL: 2.25, 3.24, 4.21

Materials: An overhead 0-99 chart or large wall chart for instructor, "Picture This" charts for participants, "What a Cut Up" cut into puzzle pieces, "What Comes Next" Activity Sheet

Time Required: 15 minutes

Directions:

1. Give each pair of participants a chart such as the Picture This! Activity Sheet. Have participants identify patterns that they see on the chart. They will probably identify the place value patterns dealing with the tens and ones places.
2. Pick a row. Add the digits of consecutive numbers, for example: 30, 31, 32, 33, 34. What pattern do you see? Does it hold in every row of the chart?
3. Now that participants have some familiarity with the chart, give out a chart that has been cut into puzzle pieces (use What A Cut Up Activity Sheet). Before allowing participants to reassemble the hundred chart have them first place the pieces on their desk. What clues do they see that indicate which pieces go together?

   Note: Participants who need assistance with this task may be given an uncut 0-99 chart to use as a base on which to place the cut out pieces.
4. Participants assemble the chart and discuss with their partner the different techniques that could be used to reassemble the chart.
5. Have participants find the piece that has the number 65 on it. Let participants reassemble the 0-99 chart beginning with the piece that has 65 on it. Did they use the same strategies?
6. The What Comes Next? Activity Sheet shows pieces of a 0-99 chart. Have participants fill in the missing numbers and follow with a discussion about the student's reasoning as they filled in numbers.

PICTURE THIS

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

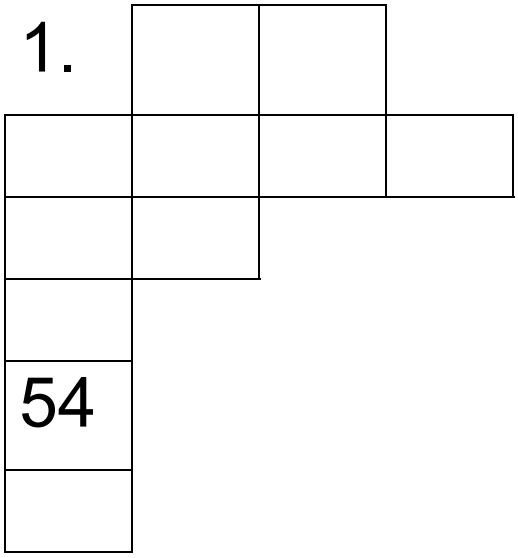
WHAT A CUT UP!
Cut on the dark lines to create puzzle pieces

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

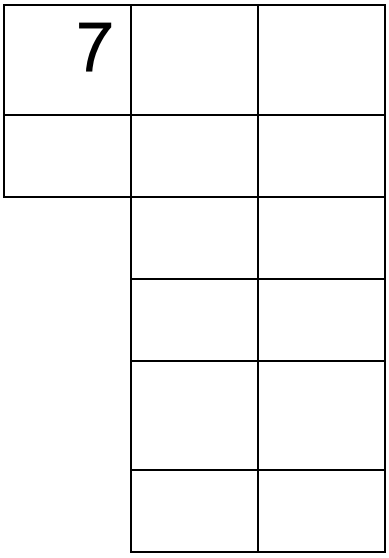
WHAT COMES NEXT?

Here are sections of a 0-99 chart. As you can see, many of the numbers are missing. Your job is to fill in the numbers to complete the section.

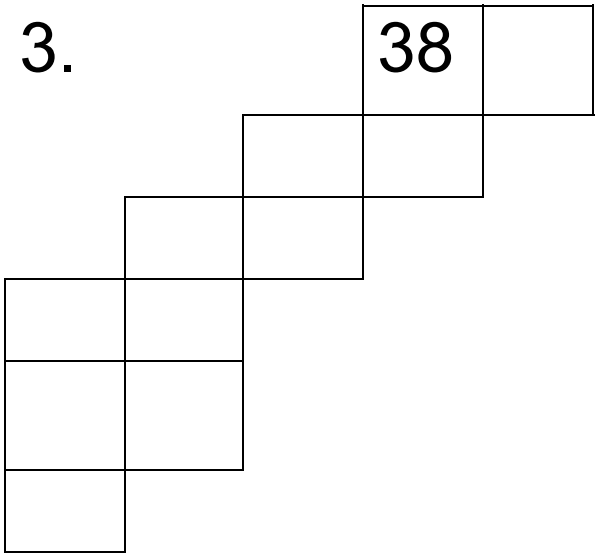
1.



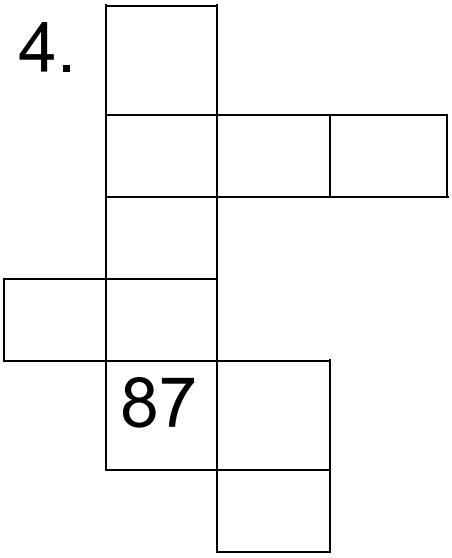
2.



3.



4.



Activity: Patterning on the 100 Chart

Format: Whole Group

Objective: Participants will be able to describe numerical and visual patterns made by skip counting on the 100 chart and connect these patterns to multiplication facts.

Related SOL: 1.21, 2.25, 3.24, 4.21, 5.20

Materials: 100 chart, linking cubes, recording sheets

Time Required: 15 minutes

Directions:

Skip counting can furnish practice with multiples while deepening students' understanding of multiplication facts. Participants are able to observe visual patterns resulting from identifying the multiples of a number.

1. Beginning with the number 2, participants will skip count by 2s marking the multiples of 2s with a linking cube. Participants will begin placing the linking cubes as they skip count, however, many participants will soon see the pattern and begin placing the cubes using the pattern rather than the skip counting. Have participants describe the pattern.
2. Participants will skip count by 3s, 4s, 5s, etc., up to 12s. It is important that participants be able to verbally describe the pattern.
3. At a later time, have participants repeat the previous activity. This time have participants record their findings on the Recording Sheet. After they have skip counted by 3, for example, ask the participants what number is under the 4th three. When they respond 12, reply that 4 threes is equal to 12. Continue this type of questioning so that participants understand the relationship between skip counting the multiples and multiplication.
4. Look at the Recording Sheet. What are the differences and similarities among the patterns? Is 239 a multiple of 6? How do you know? If a number is a multiple of 6, is it a multiple of 2? Of 4?

   Extension: Skip count by 2s and then by 3s, marking the multiples of 2 with one color and the multiples of 3 with a different color linking cube. Which numbers have two colors on them (6,12,18,...) Why?

Why is “Common Multiples” a good name for this set of numbers?

Looking at the numbers that are common multiples, which number is the Least Common Multiple?

Graphing 100 Chart Patterns

1. Look back at the recording sheets where you marked all of the multiples. Transfer these findings to the Multiplication Table worksheet.
2. We want to visually represent each group of multiples. Let's use the 3s as an example to model the process. Look back at the recording sheet where you marked the multiples of 3. The first multiple was 3, the second 6, the third 9 and so on. Record this information in a chart.

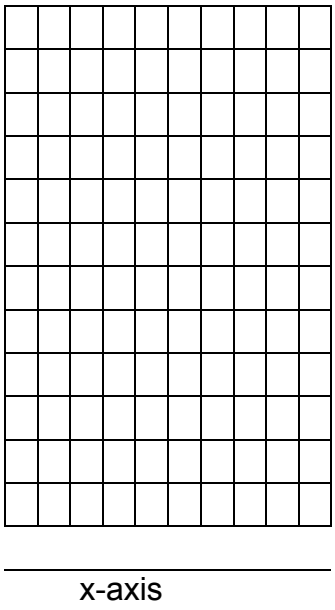
Multiples of 3	
Position of the Multiple	The Value of the Multiple
1	3
2	6
3	9
4	12
5	15

What is the relationship between the position of the multiple and the value of the multiple?

Value = 3 x position

What is the relationship between the chart and the 3 row or column on the multiplication table?

3. Graph the above information on a first quadrant graph where the position of the multiple is graphed on the x-axis and the value of the multiple is graphed on the y-axis. Lay a piece of spaghetti along the points to show that they are linear and can be connected with a straight line.



5. Continue making charts and graphing this information for the other multiples on the same graph. Compare graphs with the multiplication table.

6. Participants should look for patterns and relationships on the graph. For example, to find common multiples on the graph, look for the times tables which have points located at 12 on the y-axis. Twelve is a common multiple for those numbers. Another way to state the relationship is that all numbers on the x-axis which have points located at 12 on the y-axis are factors of 12.

Closure

Journal Entry: Explain what is meant by the term "multiple".

HUNDRED CHART

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

RECORDING SHEET

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Activity: Odd and Even

Format: Whole group

Objective: Participants will be able to represent even and odd numbers using concrete materials and make conjectures about adding and subtracting even and odd numbers.

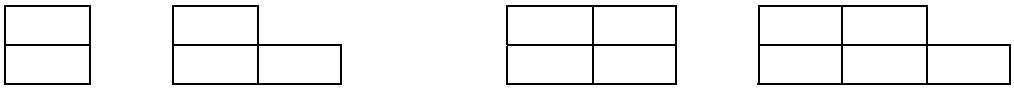
Related SOL: 2.5

Materials: color tiles

Time Required: 5 minutes

Directions:

1. Distribute color tiles to participants.
2. Participants will attempt to create rectangles where one side is two inches high using 2 tiles, then 3 tiles, 4 tiles etc. The pattern should look similar to the following:



2 **3** **4** **5**

3. What do you notice about the pattern shapes?

   Participants should notice that 2, 4, 6, 8, etc., are rectangles where all of the tiles have partners while 3, 5, 7, etc., there is a tile left over that does not have a partner. Use these models to explain why the numbers 2, 4, 6, 8, etc., are called even and 1, 3, 5, 7, etc., are called odd.

4. Model the following scenarios:

   What shape will result if you add two even numbers?

An even + an even will always equal an even number. Why?

What will result if you add two odd numbers?
What about an even and an odd number?

5. Have participants grab a handful of tiles. Count the tiles. Is this an even number or an odd number?

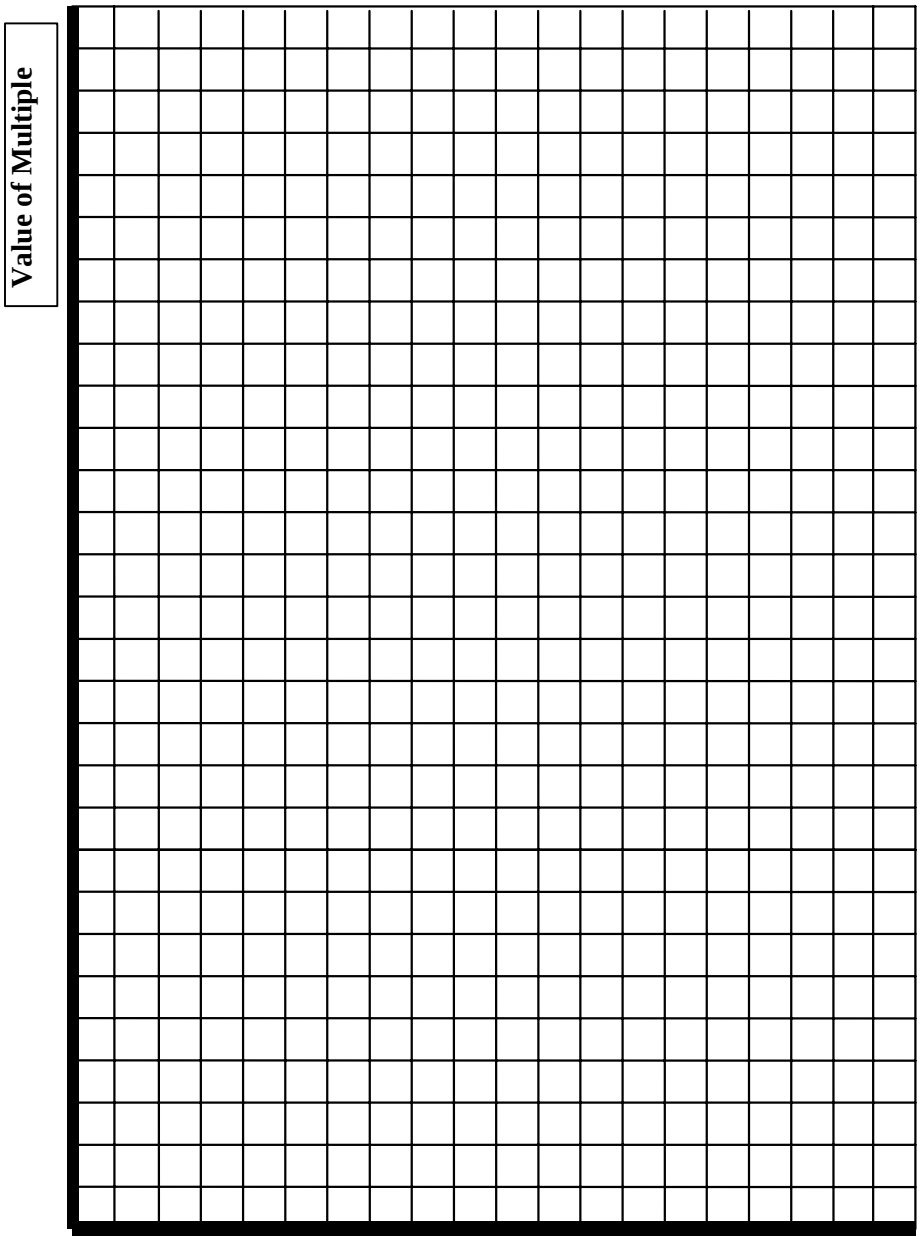
   Attempt to form a rectangle of height 2 to determine if the tiles will all have partners or if there will be an odd man out.

Multiplication Table Activity Sheet

X	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

GRAPHING HUNDRED CHART PATTERNS

Multiple of	
Position of multiple	The value of the multiple



Activity: Line Up!

Format: Whole group

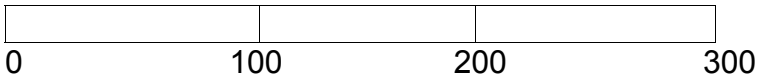
Objective: Participants will be able to identify and locate missing whole numbers on a given number line.

Related SOL: 4.21

Materials: Line Up! Activity Sheet, meter sticks, yarn, and clothes pins

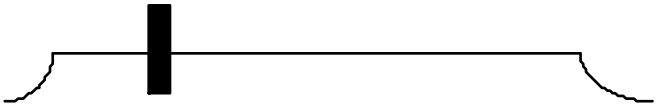
Time Required: 10 minutes

Activity A:
Tape three meter sticks to the blackboard. Participants identify points for numbers along the line and compare numbers. Play the game Guess The Number I Am Thinking About.



Activity B:
Attach yarn to the blackboard one meter apart and label the endpoints with two numbers that are multiples of 100. Place a clothespin on the yarn to represent the point in question. Participants estimate the number at the clothespin. Place a meter stick along the yarn to check the estimate.

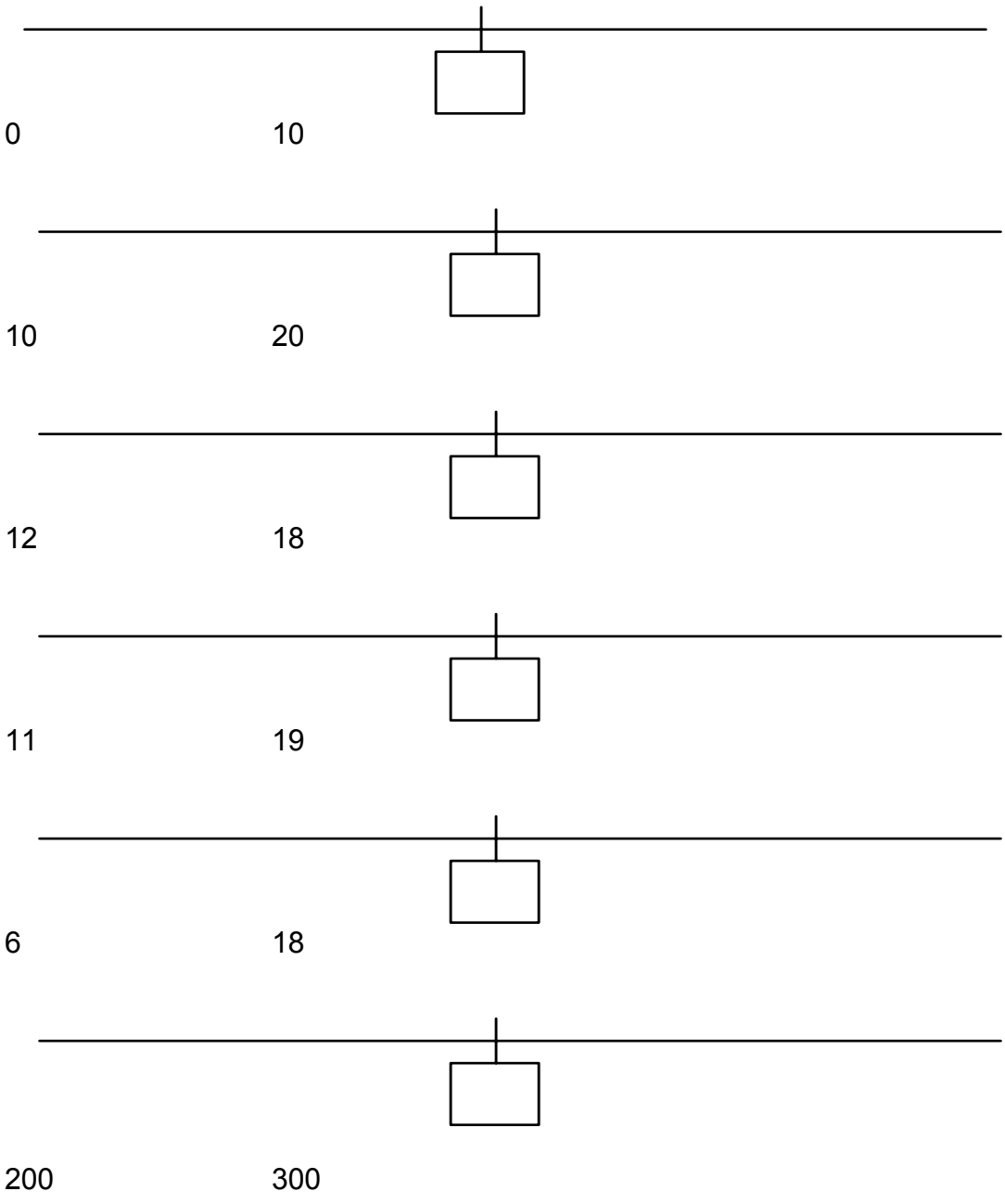
300 clothespin 400



Activity C:
The Line Up Activity Sheet gives participants practice in identifying numbers that are half way between two other numbers. Have participants discuss the strategies they used to fill in the numbers. Additional problems should be used with the missing number in different locations.

LINE UP!

Each of the following segments has a missing number. Fill in the missing numbers. Be able to give a reason for your answer.



Activity: Arrow Math

Format: Whole Group

Objective: Participants will be able to describe numerical and visual patterns made by skip counting on the hundred chart and connect these patterns to multiplication facts.

Related SOL: 1.21, 2.25, 3.24, 4.21, 5.20

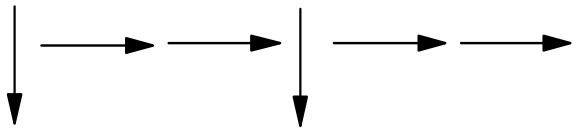
Materials: Arrow Math on the Hundred Chart Activity Sheet

Time Required: 5 minutes

Directions:

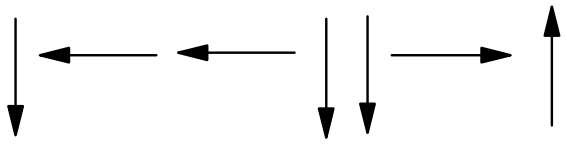
1. Have participants discuss this type of activity for developing mathematical reasoning with patterns. For example, if a student goes forward one and back one, what is the total effect; or up and down ten? Will students see the impact of arrow reversals? Will students begin to see how they can reduce the number of arrow steps by canceling out opposites? Further, ask participants how does arrow math reinforce place value concepts?

ARROW MATH ON THE HUNDRED CHART

34  **MYSTERY NUMBER: _____**

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

ARROW MATH ON THE HUNDRED CHART

28 

MYSTERY NUMBER: _____

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
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71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Activity: The King's Commissioners

Format: Whole Group

Objective: Participants will be able to recognize patterns that occur in the selection as they assist the Princess and the Royal Advisors by counting by 2s, 5s, and 10s. Participants will then be able to recognize, create, extend, and describe sequential numerical patterns on a hundreds chart.

Note: *The King's Commissioner* is a wonderful story that encourages students to examine not only place value but also the patterning that occurs in our number system. Participants are invited to count by 2s, 5s, and 10s as they attempt to solve the problems that besiege the royal kingdom. The story is mathematically sound because it reinforces the important mathematical concept that there are different ways to think about and solve the same problem. Perhaps the greatest value of the selection is in the very fact that all the thinking and pondering of the Royal Advisors and the Princess are correct just different.

Related SOL: 3.24, 4.21, 5.20

Materials: *The King's Commissioners*, hundreds boards (1 - 100), centimeter cubes, crayons or markers

Time Needed: 20 minutes

Directions:

1. Read *The King's Commissioners* by Aileen Friedman in its entirety to the class in order to describe to participants how the process of counting from 1 to 100 can be simplified with the recall of basic multiplication combinations.
2. Explain to the participants that they are going to find multiples of the numbers 2, 3, 4, 5, 6, 7, 8, 9, and 10 on a 100 chart. Remind participants that they should look for patterns on their charts and anticipate the patterns they will find.
3. Instruct participants to place a 100 chart in front of them as well as 15 centimeter cubes.
4. Explain to the participants that they are going to locate all the multiples of 2 and may begin by placing a blue cube on the number 2.

5. Ask the participants to place a blue cube on every other consecutive square on the 100 chart starting with the number 2. They may begin by orally reciting 1, 2, 1, 2. Whenever they recite 2, they should place a blue cube on the chart. They should place all 15 cubes on the numbers 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, and 30.
6. Explain to the participants that the numbers covered with blue cubes are multiples of 2. Ask participants if they notice a pattern and if they recognize other numbers on the chart that are multiples of two.
7. The participants should then color with a blue crayon or marker all the multiples of 2.
8. Participants should then be provided with another 100 chart and different color cubes.
9. Explain to the participants that they are now going to locate the multiples of 3.
10. Instruct participants that they are to use pink cubes for 3, green cubes for 4, purple cubes for 5, and so on.
11. Participants then arrange the cubes on the chart and then subsequently color the chart to show the multiples. Remind participants to always look for a pattern and attempt to extend the pattern to show all possible multiples of the number on the 100 chart.
12. Arrange all the completed charts of the multiples where the participants can make observations. Discuss the patterns that have become obvious.

Activity: *The King's Chessboard*

Format: Whole Group

Objective: Participants will be able to create a table that will allow them to see the results of exponential growth.

Note: *The King's Chessboard* by David Birch enables participants to learn the effects of doubling numbers and the pattern that is created by this geometric expansion.

Related SOL: 3.24, 4.21, 5.20

Materials: *The King's Chessboard*, 100 chart (1-100), grains of rice

Time Required: 20 minutes

Directions:

1. Begin this lesson by reading the selection *The King's Chessboard* to the class.
2. Some of the participants may already be familiar with the story of the pauper who asked the king to repay his kindness by placing a grain of rice on the king's chessboard and then doubling it for every one of the 64 squares on the board. Without thought into the matter, the king assumed that this was a small request but it ended up costing him his kingdom.
3. Explain to participants that they are going to begin the activity by putting one grain of rice on square one and then doubling this to two grains of rice, then doubling this to four grains on square three.
4. Ask participants to estimate and record how many squares they will be able to cover effectively before they run out of rice.
5. Most participants will be amazed at how quickly these numbers multiply and depending upon the grains of rice each student has, few participants will be able to continue far on the hundreds board chart.
6. From this information, have participants create a table that will show the resulting number on the 64 squares of the king's chessboard. Allow participants to use the calculator to assist in their efforts.
7. Encourage participants to attempt to write a rule and to create a formula that will allow them to deduce the last number.